BOOK OF ABSTRACTS

Maximal dissipative operators and squares of symmetric operators
Yury Arlinskiĭ

In an infinite-dimensional separable complex Hilbert space we give abstract constructions of families $\{\mathcal{T}_z\}_{\mathrm{Im}\, z>0}$ of congruent closed densely defined symmetric operators with the properties: (I) the domain of $\mathcal{T}_z^2$ is a core of $\mathcal{T}_z$, (II) the domain of $\mathcal{T}_z^2$ is dense but note a core of $\mathcal{T}_z$, (III) the domain of $\mathcal{T}_z^2$ is nontrivial but non-dense, (IV) $\mathrm{dom}\, \mathcal{T}_z^2 = \{0\}$. For this purpose a class of maximal dissipative operators is defined and studied.

Given a densely defined closed symmetric operator S, in terms of the intersection $\mathrm{dom}(S) \cap \mathrm{ran}(S - \lambda I)$ and the projection of $\mathrm{dom}(S^*)$ on $\mathrm{ran}(S - \lambda I)$, $\lambda \in \mathbb{C} \setminus \mathbb{R}$, necessary and sufficient conditions for the cases (I)–(IV) related to the domain of S^2, are obtained.

Results are published in Arlinskiĭ, Yu.M.: Families of symmetric operators with trivial domains of their squares, Complex Anal. Oper. Theory, **17**(7), 34pp. (2023) and in Arlinskiĭ, Yu.M.: Squares of symmetric operators, Complex Anal. Oper. Theory, **18**(7), 47 pp. (2024).

Spectral theory of two dimensional Laplacians with oblique Robin boundary conditions
Jussi Behrndt (Graz)

In this talk we discuss a new class of Robin type boundary conditions for the Laplacian on an interior or exterior domain in $\mathbb{R}^2$. In contrast to the classical situation, where only the normal component of the derivative appears in the boundary condition, we also add a tangential component and vary its strength with a parameter. It turns out that in some cases this additional tangential component leads to unexpected spectral properties of the corresponding self-adjoint realization of the Laplacian.

The essential numerical range for differential operators
Sabine Bögli (Durham)

The essential numerical range is a useful concept to describe spectral pollution when approximating a linear operator by projection or domain truncation methods. In this talk I will first give an introduction to the topic and then present some new results for differential operators for which a modified definition gives tighter bounds on the set of spectral pollution. This is joint work with Marco Marletta (Cardiff) and Christiane Tretter (Bern).

Spectral cluster bounds for orthonormal functions on compact manifolds with boundary
Jean-Claude Cuenin (Loughborough)

I will talk about L^q bounds for spectral clusters (linear combinations of eigenfunctions) of the Laplace-Beltrami operator on a 2-dimensional compact manifold with boundary. Smith and Sogge (2007) proved optimal bounds for the case of a single cluster. In this talk, we will consider N orthonormal clusters and seek bounds on the L^q norm of their square function (or density). This program was initiated by Frank and Sabin (2017), who extended the classical L^q bounds of Sogge (1988) in the boundaryless case. The main challenge is to obtain an optimal dependence on N (the number of functions involved). The talk is based on joint work with Xiaoyan Su and Ngoc Nhi Nguyen.

Spectral properties of dispersive and absorbing photonic crystals
Christian Engström (Linnaeus University, Växjö, Sweden)

Photonic crystals are nanostructures characterized by a periodic modulation of their material properties. Their usefulness, especially when engineered with deliberate defects, is analogous to that of electronic semiconductors.

This talk reviews recent progress in the mathematical analysis of dispersive and absorbing photonic crystals. The focus is on spectral properties for Maxwell's equations with the Drude-Lorentz model, which is used to model metal-dielectric photonic crystals.

The talk is based on joint work with Heinz Langer, Axel Torshage, and Christiane Tretter [1–3]

[1] C. Engström, H. Langer, C. Tretter *Rational eigenvalue problems and applications to photonic crystals*, Journal of Mathematical Analysis and Applications, 445 (1), 240-279, 2017.
[2] C. Engström, A. Torshage, *Accumulation of complex eigenvalues of a class of analytic operator functions*, Journal of Functional Analysis 275 (2), 442–477, 2018
[2] C. Engström, *On spectral enclosures for Maxwell's equations with the Drude-Lorentz model*, Applied Mathematics Letters, 155, 109137, 2024.

On some geodesic flows on Fréchet Lie groups
Joachim Escher (Hannover)

Of concern is a geometric approach in the sense of V. Arnold to study various model equations appearing in mathematical hydrodynamics as geodesic flows of right-invariant metrics induced by suitable Fourier multipliers on the Fréchet–Lie group of all diffeomorphisms of the n-dimensional torus and the Euclidean n-space. This method covers in particular right-invariant metrics induced by Sobolev norms of fractional order. It is shown that the corresponding initial value problem is well-posed in the smooth category and that the Riemannian exponential mapping is a smooth local diffeomorphism, provided that the symbol complies with certain mild structural conditions.

Spectral properties of non-selfadjoint Maxwell systems
Francesco Ferraresso (Verona)

Dissipative Maxwell systems find frequent application in the modelling of electromagnetic wave propagation through conductive media. The conductor absorbs part of the EM energy of the wave, resulting in loss of energy. From a mathematical point of view, conductivity makes the underlying Maxwell operator non-selfadjoint.

I will discuss a few recent results regarding the essential spectrum and the spectral approximation of dissipative Maxwell systems in unbounded domains of the three dimensional Euclidean space. Under certain assumptions on the behaviour of the coefficients at infinity, the essential spectrum decomposes in two parts, one of which is non-empty even in bounded domains. Moreover, the eigenvalues of finite multiplicity of the system can be computed exactly by means of the domain truncation method.

I will conclude with some remarks about the propagation of EM waves in semi-transparent Faraday layers.

Based on joint work with S. Bögli (Durham), M. Marletta (Cardiff), and C. Tretter (Bern).

Compact perturbations of normal operators Were are their invariant subspaces?
Eva A. Gallardo (Madrid)

In this talk, we will address the problem regarding the existence of nontrivial closed invariant subspaces of compact perturbations of normal operators acting boundedly on separable, infinite-dimensional complex Hilbert spaces. After considering the finite-rank case, we will show that a large class of such operators are decomposable, extending, in particular, recent results of Foias, Jung, Ko and Pearcy.

Decomposable operators were introduced by Foias in the sixties and many operators in Hilbert spaces are decomposable as unitary operators, self-adjoint operators or more generally normal operators. In a broad sense, decomposable operators have the most general kind of spectral decomposition possible. Consequently, every operator in the aforementioned class has a rich spectral structure and plenty of non-trivial closed invariant subspaces.

Based on joint works with F. J. González-Doña.

Energy decay for the strongly damped wave equation

Borbala Gerhat (Institute of Science and Technology Austria)

For the wave equation with damping unbounded at infinity, essential spectrum may cover the whole negative semi-axis. One can thus not expect the semigroup norm to decay exponentially in time and a more delicate analysis needs to be performed. We derive bounds for the resolvent norm (between suitable spaces) along the imaginary axis and thereby obtain the corresponding polynomial decay rates of the semigroup. This generalises a result by R. Ikehata and H. Takeda which was obtained by a different approach employing PDE analysis methods.

Based on joint work with A. Arnal, J. Royer and P. Siegl.

Weak hypocoercivity for semigroups of operators

Martin Grothaus (Kaiserslautern-Landau)

Motivated by problems from Industrial Mathematics we further developed the concepts of hypocoercivity. The original concepts needed Poincaré inequalities and were applied to equations in linear finite dimensional spaces. Meanwhile we can treat equations in manifolds or even infinite dimensional spaces. The condition giving micro- and macroscopic coercivity we could relax from Poincaré to weak Poincaré inequalities. In this talk an overview and many examples are given.

From experiment to operator theory: MHD dynamo operators—some historical comments

Uwe Günther (Helmholtz-Zentrum Dresden-Rossendorf)

The mean-field α^2-dynamo model of magnetohydrodynamics (MHD) is discussed concerning the physically relevant conditions allowing for self-exciting modes. Such over-critical dynamo regimes had been demonstrated experimentally for the first time at especially designed liquid-sodium facilities in Riga and Karlsruhe in 1999. The analysis of corresponding measurement output raised the question of possibly existing isospectral dynamo configurations. In clarifying this aspect by a no-go-theorem, the fundamental symmetry of the differential expression of the corresponding block-operator matrix played an important role — indicating possible structural links to operator theory in Krein spaces and physical models of so-called $\mathcal{PT}$ Quantum Mechanics (PTQM). Some structural parallels between α^2-dynamo operators with idealized boundary conditions and PTQM operators are briefly discussed with regard to the occurrence of spectral branch points ($\mathcal{PT}-$phase transitions). In passing to α^2-dynamo operators with physically realistic boundary conditions, the reversal mechanism of the magnetic field is sketched. For the corresponding non-self-adjoint spectral problem of a system of two singular 2nd-order differential operators global estimates of the spectrum allowed for the formulation of an anti-dynamo-theorem and a non-oscillation theorem. The conditions of these theorems have to be violated to achieve oscillatory dynamo regimes — a goal which is planned to be reached in an upcoming experiment at the large-scale dynamo facility DRESDYN. The current state of this experiment is briefly described.

This is joint work with Frank Stefani and Christiane Tretter.

Essential numerical range, essential spectra and essential norm resolvent estimates
Nicolas Hefti (Bern)

This talk has two main parts. First, I introduce a new essential spectrum $\sigma_{e5,ap}(T)$ for closed linear operators T and use it to show that the essential numerical range $W_e(T)$ contains the entire approximate point spectrum $\sigma_{ap}(T)$, except possibly for isolated eigenvalues of $\sigma_{ap}(T)$ of finite algebraic multiplicity.

Second, I present novel two-sided estimates of the essential resolvent norm. Specifically, I show that the growth of $\|(T-\lambda)^{-1}\|$ is controlled by the distance from $\lambda \in \rho(T) \setminus W_e(T)$ to $W_e(T)$.

If time permits, I will combine these results to derive new perturbation results for $\sigma_{e5,ap}(T+A)$ and other essential spectra under relatively bounded perturbations A.

Spectral Problems related to the Q-Tensor Model for Liquid Crystals
Matthias Hieber (Darmstadt)

Recent well-posedness results for the Q-Tensor model for liquid crystals are based on the property that the associated linear operator is sectorial. We show that the proof of this property within the Hilbert space setting is based on spectral theory, the numerical range and Schur complements.

Stability via closure relations
Birgit Jacob (Wuppertal)

We consider differential operators A that can be represented by means of a so-called closure relation in terms of a simpler operator A_{ext} defined on a larger space. We analyze how the spectral properties of A and A_{ext} are related and give sufficient conditions for exponential stability of the semigroup generated by A in terms of the semigroup generated by A_{ext}. As applications we study the long-term behaviour of a coupled wave-heat system on an interval.

The spectrum of the Diracoperator on compact quotients of the oscillator group
Margarita Kraus (Mainz)

The subject of the talk is a contribution to the analysis of Diracoperators on locally symmetric Lorentzian manifolds. Specifically, we study a one parameter family of Diracoperators D_t on manifolds of the form $\Gamma\backslash\mathrm{Osc}$ where Osc is the four-dimensional oscillator group and Γ a lattic in it. We use representation theory to determine the spectrum of the Diracoperators D_t.

Higher Dimensional Fourier Quasicrystals from Lee-Yang Varieties
Pavel Kurasov (Stockholm)

Fourier Quasicrystals (FQ) are defined as crystalline measures

$$\mu = \sum_{\lambda \in \Lambda} a_\lambda \delta_\lambda, \quad \hat{\mu} = \sum_{s \in S} b_s \delta_s,$$

so that not only μ (and hence $\hat{\mu}$) are tempered distributions, but also

$$|\mu| := \sum_{\lambda \in \Lambda} |a_\lambda| \delta_\lambda \quad \text{and} \quad |\hat{\mu}| := \sum_{s \in S} |b_s| \delta_s,$$

are tempered.

One-dimensional FQs with positive integer weights (that is $a_\lambda \in \mathbb{N}$) can be described using stable Lee-Yang polynomials, as was proven in a joint work with Peter Sarnak. Multidimensional Fourier quasicrystals are discussed in the current talk. It is shown that a rather general family of FQs in $\mathbb{R}^d$ with positive integer weights can be constructed using co-dimension d Lee-Yang varieties in $\mathbb{C}^n$, $n > d$. These complex algebraic varieties are symmetric and avoid certain regions in $\mathbb{C}^n$, thus generalising zero sets of Lee-Yang polynomials.

It is shown that such FQs can be supported by Delaunay almost periodic sets and are genuinely multidimensional in the sense that their restriction to any one-dimensional subspace is not given by a one-dimensional FQ. Connections to alternative recent approaches by Yves Meyer, Lawton-Tsikh and de Courcy-Ireland-K. are clarified. Is it possible that our construction gives all multidimensional FQs with positive integer masses?

This is joint work with L. Alon, M. Kummer, and C. Vinzant (*Inventiones Mathematicae*, **39**, pp. 321–376, (2025)).

Basis properties of components of eigenvectors of operator matrices
Matthias Langer (Strathclyde)

In this talk I shall consider eigenvalues and eigenvectors of certain 2×2-operator matrices. The main focus will be on different types of basis properties of components of eigenvectors, where I am particularly interested in Riesz bases, Bari bases and p-bases.

Eigenvalues as singularities of analytic functions
Annemarie Luger (Stockholm)

Isolated eigenvalues of a selfadjoint operator A in a Hilbert space are obviously poles of the resolvent $(A-z)^{-1}$. In this talk we revisit other situations, when eigenvalues can be characterized as singularieties of some corresponding analytic function, up to very recent results (with J.Reiffenstein).

Computing spectra of (perturbed) periodic operators: how hard can it be?
Marco Marletta (Cardiff)

In this talk I shall consider the current state of numerics for the approximation of spectra of periodic and perturbed-periodic Schrödinger and related operators on infinite domains in $\mathbb{R}^d$, with particular attention to the non-selfadjoint case. I shall discuss five particular special cases, in ascending level of intractability, and illustrate the difficulties both with numerical results and with theorems that give the Hansen *Solvability Complexity Indices* (SCI) of the problems. This talk reviews work I have undertaken over the last six or seven years with co-authors including Salma Aljawi, Sabine Bögli, Jonthan Ben-Artzi, Frank Rösler and Christiane Tretter. I shall mention problems that remain open and discuss problems that, despite having SCI $= 1$, are nevertheless very challenging in practice for dimension $d \geq 2$.

Spectral theory of regular quadratic differential operator pencils
Manfred Möller (Johannesburg)

Consider boundary value problems for (second order ordinary) differential equations, e.g.,

$$-y'' + p_1(\cdot, \lambda)y' + p_2(\cdot, \lambda)y = 0$$

with boundary conditions

$$B_{11}(\lambda)y(a) + B_{12}(\lambda)y'(a) = 0, \;\; B_{21}(\lambda)y(b) + B_{22}(\lambda)y'(b) = 0,$$

where $p_1(\cdot, \lambda)$ and $p_2(\cdot, \lambda)$ are polynomials of degree ≤ 1 and 2, respectively, with sufficiently regular coefficient functions on $[a, b]$, whereas the B_{rs} are polynomials of degree at most 2 with complex coefficients. We will give an overview on properties of the spectra of regular problems (problems with spectra consisting of infinitely many discrete eigenvalues of finite multiplicity). The focus will be on problems which can be represented by an operator polynomial

$$L(\lambda) = \lambda^2 M - i\lambda K - A$$

with self-adjoint operators M, K, A. The spectrum is not necessarily real, but is still symmetric with respect to the imaginary axis. The eigenvalues, with multiplicity, are the roots of the characteristic function of the eigenvalue problem, with well-defined asymptotic properties—the direct problem. Conversely, eigenvalues for reasonably large classes then give information on the given data of the eigenvalue problem—the inverse problem.

Consequently, the direct spectral problem can be split into two tasks: firstly, to find the characteristic determinant, and secondly, to find the zeros of the characteristic determinant.

Hence classes $\mathfrak{D}$ of differential operators, $\mathfrak{A}$ of entire functions, and $\mathfrak{E}$ of infinite unordered tuples of complex numbers will be considered in order to investigate the following questions.

1. Is there a bijective correspondence between the differential operators in $\mathfrak{D}$ and their characteristic determinant as functions in $\mathfrak{A}$?

2. Is there a bijective correspondence between the entire functions in $\mathfrak{A}$ and (the sequences) of their zeros as elements in $\mathfrak{E}$?

Non-selfadjoint spectral problems related to self-similar blowup in nonlinear wave equations
Kaori Nagatou (Karlsruhe)

We consider the wave equation with a power nonlinearity

$$u_{tt} - \Delta u = u^p,$$

with initial profiles $u(x,0)$ and $u_t(x,0)$, $x \in \mathbb{R}^3$, $t \geq 0$, and $p > 1$ an odd integer. In order to investigate the blowup dynamics we look for radial self-similar blowup solutions of the form

$$u(x,t) = (T-t)^{-\frac{2}{p-1}} U\left(\frac{|x|}{T-t}\right), \quad T > 0,$$

with a smooth, radial profile U. In particular, we are interested in stability properties of such solutions. This gives rise to analyzing the spectrum of the linearized operator, i.e. to the non-selfadjoint eigenvalue problem:

$$\mathcal{L}\mathbf{u} = \lambda\mathbf{u},$$

where $D(\mathcal{L}) \subset H^2_{\text{rad}}(B^3) \times H^1_{\text{rad}}(B^3)$, $H^k_{\text{rad}}(B^3) := \{\mathbf{u} \in H^k(B^3) : \mathbf{u} \text{ is radial}\}$,

$$\mathcal{L}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} := \begin{pmatrix} -\rho u_1'(\rho) - \alpha u_1(\rho) + u_2(\rho) \\ u_1''(\rho) + \frac{2}{\rho}u_1'(\rho) - \rho u_2'(\rho) - (\alpha+1)u_2(\rho) + V(\rho)u_1(\rho) \end{pmatrix},$$

$B^3 = \{x \in \mathbb{R}^3 : |x| \leq 1\}$, $\rho = \frac{|x|}{T-t}$, $V(\rho) = pU(\rho)^{p-1}$ and $\alpha = \frac{2}{p-1}$. We are interested in excluding eigenvalues of $\mathcal{L}$ in the crucial parts of the right complex half plane, which is ongoing work together with B. Schörkhuber, Y. Watanabe, M. Plum and M.T. Nakao. We first provide a compact set $R \subset \mathbb{C}$ such that no eigenvalues can exist in the right half-plane outside this set. Furthermore, we derive estimates ensuring that small discs with explicitly computable radii and carefully chosen centers are free of eigenvalues. Covering the set R with such discs gives, in principle, the desired non-existence of eigenvalues, implying (linear) stability.

Computer-assisted Existence Proofs for Navier-Stokes Equations on an Unbounded Strip with Obstacle
Michael Plum (Karlsruhe)

The incompressible stationary 2D Navier-Stokes equations are considered on an unbounded strip domain with a compact obstacle, and with the Poiseuille flow as a background flow near infinity. A computer-assisted existence and enclosure result for the velocity (in a suitable divergence-free Sobolev space) is presented, based on Newton-Kantorovich-like arguments. Starting from an approximate solution (computed with divergence-free finite elements), we determine a bound for its defect, and a norm bound for the inverse of the (non-selfadjoint) linearization at the approximate solution, which we obtain via eigenvalue bounds (supplemented by bounds on the essential spectrum) for some auxiliary self-adjoint eigenvalue problem. For computing these eigenvalue bounds, we use the Rayleigh-Ritz method, the Temple-Lehmann-Goerisch method, and a homotopy method for obtaining the needed spectral pre-information. In detail, three homotopies are performed; the first deforms the coefficients into piecewise constant ones, the second deforms the domain with obstacle into the full strip, and the third fades out the divergence condition.

This is joint work with Jonathan Wunderlich, Karlsruhe Institute of Technology.

Computing eigenvalues of the Klein-Gordon equation Frank Rösler

We construct a new numerical method for computing isolated eigenvalues of the Klein-Gordon equation with potential V satisfying the hypothesis

$$\|V\|_{W^{1,p}(\mathbb{R}^d)} \leq M \quad \text{ and } \quad |V(x)| \leq M(1+|x|^2)^{-\frac{1}{2}} \text{ for all } x \in \mathbb{R}^d$$

for some $p > d$. The spectral problem for the Klein-Gordon equation consists of finding a pair $(u, \lambda) \in L^2(\mathbb{R}^d) \times \mathbb{C}$ such that $T_V(\lambda)u := (-\Delta + m^2 - (V - \lambda)^2)u = 0$, where $m > 0$ is a parameter. The set of eigenvalues of the quadratic operator pencil T_V is denoted $\sigma_p(T_V)$.

Based on results from [Langer-Tretter (2006)], we define a sequence of computer algorithms Γ_n, which accept finitely many point values of V and produce computable subsets $\Gamma_n(V) \subset \mathbb{C}$. We prove that (i) if the hypothesis above holds, then $\Gamma_n(V) \to \sigma_p(T_V)$ in Hausdorff distance and (ii) if the constant M is known a-priori, we provide explicit error bounds from above. Our results have implications for the so-called Solvability Complexity Index (SCI) Hierarchy [Ben-Artzi et al. (2015)]: the spectral problem for the Klein-Gordon equation is (i) in the class Δ_2 if the above hypothesis holds (i.e. the spectrum can be computed in one single limit) and (ii) in the class Π_1 if additionally the constant M is known a-priori.

The talk is based on joint work with Christiane Tretter (Bern).

How to calculate the eigenvalues of the spheroidal wave equation Harald Schmid (Amberg-Weiden)

The (angular) spheroidal wave equation plays an important role in many fields of physics and engineering like quantum mechanics, electromagnetism, signal processing etc. We are looking for an easy-to-use numerical method to determine the eigenvalues and the corresponding eigenfunctions of the spheroidal wave equation. For this purpose, an associated linear differential system with two regular singular points is studied. An explicit formula for the connection coefficients between its Floquet solutions results in an entire function whose zeros are exactly the spheroidal eigenvalues we are looking for. This entire function can be calculated by means of a recursion formula with arbitrary accuracy and rather small computational effort. This new approach seems to be more appropriate for practical calculations than previous methods, and it also offers a new insight into the spectrum of the spheroidal wave equation.

The Plasmonic Eigenvalue Problem and the Dirichlet-to-Neumann Operator on Manifolds with Fibered Cusps
Elmar Schrohe (Hannover)

A plasmon of a bounded domain $\Omega \subseteq \mathbb{R}^n$ is a nontrivial harmonic function on $\mathbb{R}^n \setminus \partial\Omega$ that is continuous at $\partial\Omega$ and whose interior and exterior normal derivative at $\partial\Omega$ have a constant ratio. This ratio is called a plasmonic eigenvalue of Ω. It is indeed an eigenvalue of $N_+^{-1}N_-$, where $N_\pm$ denote the exterior and interior Dirichlet-to-Neumann operators.

Motivated by the case of two touching convex domains in $\mathbb{R}^n$, we consider this problem on a manifold with fibered cusp singularities. In a first step we show that the Calderòn projector for elliptic operators in this setting is a matrix of ϕ-pseudodifferential operators in the sense of Mazzeo and Melrose. From this we derive that also the Dirichlet-to-Neumann operator is a first order ϕ-pseudodifferential operator. This gives us a precise understanding of the behavior of its Schwartz kernel near the boundary.

Joint work with Karsten Fritzsch and Daniel Grieser.

Optimal version of the fundamental theorem of chronogeometry
Peter Semrl (Ljubljana)

The fundamental theorem of chronogeometry describes the general form of mappings on classical Minkowski space that preserve lightlikeness in both directions. This theorem has been generalized in several ways, and the optimal improvement will be presented.

Local form subordination without a power decay and the Riesz property of spectral projections
Petr Siegl (Graz)

We revisit the local form subordination condition on the perturbation of a self-adjoint operators with compact resolvent. This condition relates the size of gaps between the unperturbed eigenvalues with the strength of perturbation and it is used to establish the Riesz basis property of the eigensystem of the perturbed operator. Our new approach allows for a slow and non-monotone decay in the subordination condition as well as for a general behavior of unperturbed eigenvalues. The abstract results are applied in Schroedinger operators with possibly unbounded or singular complex potential perturbations.

The talk is based on joint works with B. Mityagin (OSU, USA).

On the spectral pair for Schrödinger operators with complex potentials on the half-line
František Štampach (Prague)

We introduce a new operator-theoretic concept of spectral pair defined for Schrödinger operators H with complex-valued bounded potentials on the half-line. The spectral pair consists of a measure ν and a complex-valued function ψ. We demonstrate that, from a perspective of inverse spectral theory, the spectral pair generalises the classical spectral measure to the non-self-adjoint case. First, we establish injectivity of the spectral map $H \mapsto (\nu, \psi)$, i.e. a variant of the Borg-Marchenko uniqueness theorem. Second, we derive asymptotic formulas for the spectral pair, in the spirit of Marchenko. In the case of real-valued potentials, we relate the spectral pair to the spectral measure of H. The talk is based on a joint work with A. Pushnitski.

The QNR and Christiane
Carsten Trunk (Ilmenau)

I will say some words about the Quadratic Numerical Range (QNR) for block operator matrices, which is one of Christiane's main research topics. This will be complemnented by some remarks on our joint research collaboration.

If time permits, I will add some nice words about Christiane :).

Iterative energy reduction for the numerical solution of nonlinear variational PDE
Thomas Wihler (Bern)

Critical points of energy functionals – of broad interest in areas such as physics and mechanics – arise as solutions to the associated Euler-Lagrange equations. While classical computational methods for approximating such models typically focus solely on the underlying partial differential equations, we propose an approach that also incorporates the energy structure itself. Specifically, we examine (linearized) iterative Galerkin discretization schemes that ensure energy reduction at each step. Additionally, we provide necessary conditions, which are applicable to a wide range of problems, that guarantee convergence to critical points of the PDE.

Computing the Quadratic Numerical Range

Christian Wyss (Wuppertal)

We present a new algorithm to compute the quadratic numerical range of a matrix. It performs significantly better than random vector sampling, the canonical method used so far.

The quadratic numerical range was introduced by Heinz Langer and Christiane Tretter in 1998. It is a subset of the usual numerical range that still contains the spectrum. While the non-convexity of the quadratic numerical range is one of its advantages over the numerical range as it allows for tighter spectral enclosures, it is at the same time one of the reasons that make the quadratic numerical range hard to compute numerically. We describe how our algorithm overcomes these difficulties. A key ingredient is a steepest ascent method with a penalty term to locate non-convex parts of the boundary. We also present a theoretical result that explains why random vector sampling becomes infeasible already for moderately sized matrices.

The talk is based on joined work with Birgit Jacob and Lukas Vorberg.